Proposal of Syntax and Semantics of CTL for MCC'16

Lasse Steen Jensen, Isabella Kaufmann, Peter Fogh, Samuel Pastva, Søren Moss Nielsen, Tobias Skovgaard Jepsen, and Jiri Srba
{lasjen12,ikaufm12,pfogh12,spastv16,smni12,tjeps12}@student.aau.dk and srba@cs.aau.dk

Department of Computer Science, Aalborg University,
Selma Lagerlöfs Vej 300, 9220 Aalborg East, Denmark

1 Petri Nets

A Path in a Petri Net A path π starting in a marking M is a finite or infinite sequence of markings and transition firings, written as

$$M = M_0 \to M_1 \to M_2 \to \dots$$

A maximal path is defined as a path that is either infinite or ends in a marking M_i such that $M_i \not\to$; also called a deadlock. The set of all maximal paths for a Petri net N from the marking M is denoted $\Pi_{max}(M)$.

2 Computation Tree Logic

$$\varphi ::= \mathit{true} \mid \mathit{false} \mid \textbf{is\_fireable}(Y) \mid \psi_1 \bowtie \psi_2 \mid \neg\varphi \mid \varphi_1 \wedge \varphi_2 \mid \varphi_1 \vee \varphi_2 \mid$$
$$EG\ \varphi \mid AG\ \varphi \mid EF\ \varphi \mid AF\ \varphi \mid EX\ \varphi \mid AX\ \varphi \mid E\varphi_1 U \varphi_2 \mid A\varphi_1 U \varphi_2$$

$$\psi ::= \psi_1 \oplus \psi_2 \mid c \mid \textbf{token\_count}(X)$$

Here $\bowtie \in \{\leq, \leq, =, \geq, \geq\}$, $X \subseteq P$, $Y \subseteq T$, $c \in \mathbb{N}^0$ and $\oplus \in \{+, -, \cdot\}$. The semantics of a CTL formula φ over a given marking M of the Petri net N is defined in Table 1. Function $eval_M$ is defined recursively in Table 2. The rest of the operators is defined as abbreviations in Table 3.

$M \models true$	
$M \models \neg\varphi$	iff $M \not\models \varphi$
$M \models \varphi_1 \wedge \varphi_2$	iff $M \models \varphi_1$ and $M \models \varphi_2$
$M \models EX\ \varphi$	iff $\exists M' : M \rightarrow M'$ and $M' \models \varphi$
$M \models E\varphi_1 U \varphi_2$	iff $\exists(M = M_0 \rightarrow M_1 \rightarrow M_2 \rightarrow \ldots) \in \Pi_{max}(M)$ s.t. $\exists i \in \mathbb{N}^0(M_i \models \varphi_2 \wedge \forall\ 0 \leq j < i : M_j \models \varphi_1)$
$M \models A\varphi_1 U \varphi_2$	iff $\forall(M = M_0 \rightarrow M_1 \rightarrow M_2 \rightarrow \ldots) \in \Pi_{max}(M)$ s.t. $\exists i \in \mathbb{N}^0(M_i \models \varphi_2 \wedge \forall\ 0 \leq j < i : M_j \models \varphi_1)$
$M \models \textbf{is\_fireable}(Y)$	iff $\exists t \in Y$ and $\exists M'$ s.t. $M \xrightarrow{t} M'$
$M \models \psi_1 \bowtie \psi_2$	iff $eval_M(\psi_1) \bowtie evla_M(\psi_2)$

Table 1. CTL Semantics

$eval_M(c)$	$= c$
$eval_M(\textbf{token\_count}(X))$	$= \sum_{p \in X} M(p)$
$eval_M(e_1 \oplus e_2)$	$= eval_M(e_1) \oplus eval_M(e_2)$

Table 2. $eval_M$ semantics

$$\varphi_1 \vee \varphi_2 \equiv \neg(\neg\varphi_1 \wedge \neg\varphi_2)$$
$$AX\ \varphi \equiv \neg EX\ \neg\varphi$$
$$EF\ \varphi \equiv E\ true\ U\varphi$$
$$AF\ \varphi \equiv A\ true\ U\varphi$$
$$EG\ \varphi \equiv \neg AF\ \neg\varphi$$
$$AG\ \varphi \equiv \neg EF\ \neg\varphi$$

Table 3. Standard abbreviations